DISCRETE MATH: COMBINATORICS: COUNTING, PERMUTATIONS, AND COMBINATIONS

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Probability calculations are important in many endeavors. A meteorologist for example, must calculate the probability of rain, a lottery commission must calculate the probability a winner will win and a medical researcher must calculate the probability that the results of a test are accurate.

Many probability calculations require knowing the number of ways in which an event can happen, such as the number of ways a lottery player can fill out a lottery card.

Often the numbers involved are quite large and careful methods must be used to be sure counting is done properly. However the best way to begin your work is by considering some situations involving relatively small numbers.

1. Definition

Before we can do these problems we have to have some definitions.
Combinatorics Combinatorics is a branch of mathematics concerning the study of finite or countable discrete structures.

probability Probability is the branch of mathematics that studies the possible outcomes of given events together with the outcomes' relative likelihoods and distributions.

combination The number of ways of picking k unordered outcomes from n possibilities.

event An event is a certain subset of a probability space. Events are therefore collections of outcomes on which probabilities have been assigned.

independent event Two events A and B are called independent if their probabilities satisfy $P(AB) = P(A)P(B)$ we also say that the event happens with replacement.

dependent event Two events are called dependent if their probability of B changes from the probability of A we say that the event happens without replacement.

Bag Example
In this example we want to know the probability of pulling a green ball out of a bag that has 2 blue balls, 3 white balls, and 3 red balls.
SOLUTION
independent: $\left(\frac{4}{8}\right)\left(\frac{4}{8}\right) = \left(\frac{16}{64}\right) = \left(\frac{1}{4}\right)$
dependent:$\left(\frac{4}{8}\right)\left(\frac{3}{7}\right) = \left(\frac{12}{56}\right) = \left(\frac{3}{14}\right)$

Lottery Example
If in a lottery drawing there are three numbers with possibility of each number being $0 - 9$ how many possible combinations are there?
SOLUTION
$\_ \cdot \_ \cdot \_ = 10 \cdot 10 \cdot 10 = 1000$
The probability that you chose the right number is: $\left(\frac{1}{10}\right)\left(\frac{1}{10}\right)\left(\frac{1}{10}\right) = \left(\frac{1}{1000}\right)$

A **permutation** is used to describe a counting procedure in which order matters (independent events).

There are two commonly used symbolic expressions for a permutation of three things selected from a group of five. $P(5,3)$ or ${}_5P_3$. Either of these can be evaluated by solving the expression $\frac{5!}{(5-3)!}$ or this can be solved on a calculator using a special permutation function which is located by pressing the MATH button and scrolling right to the PROB tab and then scrolling down to the nPr tool.

Generally, $P(n,r)$ is calculated by evaluation the expression $\frac{n!}{(n-r)!}$.

To solve these problems algebraically we must understand what a factorial(!)

is. The basic rules of factorials are $0! = 1$ and $1! = 1$ for $n > 1$ multiply every number together from one to n.

For example:
$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$
$6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$

We will never have a permutation or probability that has a numerator smaller than 0 or larger than the denominator, which implies that the probability will be between 0 and 1 **inclusively**.

Inclusive means that the numbers are included in the range of the probability.

Exclusive means that the range of the numbers does not include the end points.

So if we are looking for the number of telephone number combinations in a 7 digit telephone number with numbers from 0 to 9 inclusively? Exclusively?

SOLUTION

Inclusive case: The inclusive case means that the numbers 0 through 9 are included in the numbers you can choose from when finding the number of combinations. So there are 10 possibilities for the first position, 10 for the second, 10 for the third, 10 for the fourth, 10 for the fifth, 10 for the sixth and 10 for the seventh position. So we use the multiplication principle to find the total number of possibilities. So $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$ gives us $10,000,000$ possible different telephone numbers.

Exclusive case: The exclusive case does not include the end points, so you have 8 possible numbers for each of the seven digits of the phone number. So we use the multiplication principle again to find that $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = 2,097,152$ possible combinations.

Examples

(1) Which is equivalent to P(10,4), 10!/4!, or 10!/6!? Find the value of P(10,4)

(2) A multi-speed bicycle has a chain that can be moved to change the bicycle's speed. The rider uses the bicycle's front and rear shift mechanisms to move the chain from one front or rear sprocket to another. If a bicycle has three front sprockets and five rear sprockets, how many speeds does it have?

(3) Some states have vehicle license numbers that consist of three letters followed by three digits. Often the letters I, O and Z are not used because they can be confused with the numerals 1, 0, and 2, respectively.
a. If these restriction apply and if characters may be repeated, how many different license plates are possible?

b. What is the probability that a vehicle selected at random will have a license number that begins with CAT

(4) *a*. In how many ways can the coach of a baseball team arrange the battling order of nine starting players?

b.A sportscaster one suggested that a baseball team try every possible battling order for its nine starters in order to determine which one day of the week with a different batting order in each game, how long will it take to complete the experiment?

(5) *a*. In how many ways can a person draw 2 cards from a standard 52-card deck if the first card is returned to the deck before the second card is drawn?

b.In how many ways can 2 cards be drawn if the first card is not put back?